Derivatives as Functions

In the homework assigned for today you explored the relationship between the graph of a function and the graph of its derivative. From these examples, we draw the following conclusions.

> Whenever the derivative *f'(x)* is positive, the function *f(x)* increases as *x* increases.
> Whenever the derivative *f'(x)* is negative, the function *f(x)* decreases as *x* increases.

You should be able to explain this relationship based on the definitions of derivative and slope.

Higher Derivatives

When we take the derivative of a derivative (with respect to *x*) we have the **second derivative**. When we take the derivative of the second derivative we have the **third derivative.** And so on.

Notation: The most common ways of writing higher derivatives are shown in the first three columns of the table.

Function	y	$f(x)$	s	displacement
First derivative	$\frac{dy}{dx}$	$f'(x)$	$v = \frac{ds}{dt}$	velocity
Second derivative	$\frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d^2y}{dx^2}$	$f''(x)$	$a = \frac{d^2s}{dt^2}$	acceleration
Third derivative	$\frac{d}{dx}\left(\frac{d^2y}{dx^2}\right) = \frac{d^3y}{dx^3}$	$f'''(x)$		

Examples 1-3 on pages 109-110 show how to find higher derivatives. Skip Example 4.

An important application of the second derivative occurs in rectilinear motion. As the last two columns of the table indicate, velocity is the first derivative of the displacement function, and acceleration is the second derivative. See Example 5 on page 111.

Graphical Interpretation of the Second Derivative

Whenever the second derivative *f''(x)* is positive, the derivative *f'(x)* increases as *x* increases and the graph of the function *f(x)* is concave up.

Whenever the second derivative *f''(x)* is negative, the derivative *f'(x)* decreases as *x* increases and the graph of the function *f(x)* is concave down.

We apply to concept of concavity in the next lesson on curve sketching.

Exercises: Pages 111-112: 3, 9, 11, 13, 17, 21, 23, 29, 31, 37, 39